A NONSTATIONARY METHOD FOR MEASURING HEAT FLUXES

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A nonstationary method for measuring heat fluxes using a hollow cylindrical probe placed in the critical cross section of a nozzle is proposed. The temperature at a fixed distance from the inner surface of the probe is measured as a function of time. The time variation of the heat flux is determined by solving the one-dimensional nonlinear heat-conduction equation.

The most widely used nonstationary method for determining heat fluxes is the calorimeter method with a linear characteristic. In this method a uniform heat flux enters the front face of a probe in the form of a finite-sized plate with thermally insulated side and rear surfaces. By measuring the temperature of the rear surface which varies linearly with time the heat-flux density can be determined [1, 2]. The heat flux can also be determined by solving the one-dimensional linear nonstationary heat-conduction problem for a probe of specific configuration [3]. More accurate results which can be used to analyze the nature of heat transfer between a high-temperature gaseous medium and the surface of the probe can be obtained by solving the one-dimensional nonlinear nonstationary heat-conduction equation. This method is particularly suitable for high heat fluxes when the thermophysical properties of the probe are strongly temperature dependent.

We have measured the heat flux in the critical cross section of a nozzle through which a high-temperature gas is flowing. A cylindrical copper washer placed in the critical cross section of the nozzle and thermally insulated from the rest of the nozzle was used as a probe. The temperature at a fixed distance from the inner surface of the probe was measured as a function of the time. By solving the one-dimensional nonlinear heat-conduction equation the time variation of the heat flux was determined.

Figure 1 shows a schematic diagram of the probe. Thermocouple 1 in the side wall of the copper washer measures the temperature at a fixed distance from the inner surface of the probe. Thermocouple 2 is a monitor. The thermocouple readings as a function of time were recorded on a type N-700 loop oscillograph. Figure 2 shows the temperature as a function of time at a distance of 0.5 mm from the inner surface of the probe (curve 1) and on the rear surface (curve 2).

The method developed for determining the heat flux is based on the solution of the nonlinear heat-conduction equation

$$\rho_0 c(t) \frac{\partial t}{\partial \tau} = \frac{1}{r} \frac{\partial}{\partial r} \left(\lambda(t) r \frac{\partial t}{\partial r} \right) \quad (R_1 < r < R_2,\ \tau > 0) \tag{1}$$

with the boundary conditions

$$t(r_1 \tau)_{\tau=0} = t_0 \quad (R_1 \leqslant r \leqslant R_2), \tag{2}$$

$$t(r_1 \tau)_{r=R_1} = \varphi_1(\tau) \quad (\tau > 0), \tag{3}$$

$$t(r_1 \tau)_{r=R_2} = \varphi_2(\tau) \quad (\tau > 0). \tag{4}$$

It is assumed that the specific heat and thermal conductivity are functions of the temperature.

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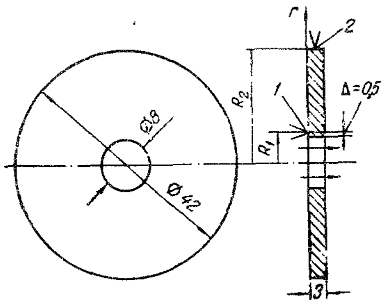

Fig. 1

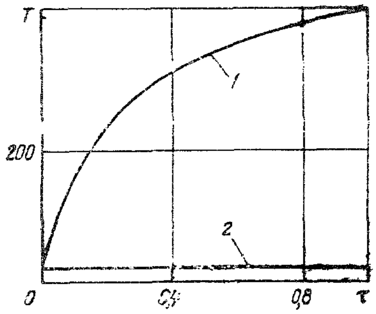

Fig. 2

Fig. 1. Schematic diagram of probe for measuring heat fluxes. 1) Location of thermocouple in side wall of probe; 2) monitoring thermocouple.

Fig. 2. Time variation of probe temperature: 1) temperature at 0.5 mm from the inner surface of the probe; 2) temperature of rear surface of probe. T in °C and τ in sec.

To obtain the temperature distribution corresponding to the solution of Eqs. (1)-(4) we consider the following expressions:

$$\Theta_1 (r_1\tau) = \frac{1}{\lambda_0} \int_{t_0}^{t} \lambda (t)\, dt, \tag{5}$$

$$\Theta_2 (r_1\tau) = \frac{1}{\rho_0 c_0} \int_{t_0}^{t} \rho_0 c (t)\, dt, \tag{6}$$

$$\Theta_3 (r_1\tau) = \frac{1}{\lambda_0} \int_{t_0}^{t} \lambda (t)\, dt + \frac{1}{\rho_0 c_0} \int_{t_0}^{t} \rho c (t)\, dt. \tag{7}$$

Differentiating Eqs. (5)-(7) once with respect to τ and twice with respect to r and rearranging, we obtain

$$\frac{\lambda_0}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Theta_3}{\partial r} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left(r (\lambda) \frac{\partial t}{\partial r} + \lambda_0 \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Theta_2}{\partial r} \right) \right), \tag{8}$$

$$\rho_0 c_0 \frac{\partial \Theta_3}{\partial \tau} = \rho_0 c (t) \frac{\partial t}{\partial \tau} + \rho_0 c_0 \frac{\partial \Theta_1}{\partial \tau}. \tag{9}$$

Subtracting (8) from (9) and using (1) we can write

$$\frac{\partial \Theta_3}{\partial \tau} - a_0 \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Theta_3}{\partial r} \right) = \frac{\partial \Theta_1}{\partial \tau} - a_0 \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Theta_2}{\partial r} \right), \tag{10}$$

and using (1) it follows from (5) and (6) that

$$\frac{\partial \Theta_2}{\partial \tau} = a_0 \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Theta_1}{\partial r} \right). \tag{11}$$

After differentiating Eqs. (10) and (11), respectively, once with respect to τ and twice with respect to r and replacing Θ_2 by Θ_1, we can write Eq. (10) in the form

$$\frac{\partial^2 \Theta_3}{\partial \tau^2} - a_0 \frac{\partial}{\partial \tau} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Theta_3}{\partial \tau} \right) \right] = \frac{\partial^2 \Theta_1}{\partial \tau^2} - a_0^2 \frac{\partial^2}{\partial r^2} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Theta_1}{\partial r} \right) \right]. \tag{12}$$

It was calculated in [4] that linear heat-conduction equations higher than second-order ensure the determination of the temperature distribution to an accuracy sufficient for practical purposes.

Consequently, it can be assumed that

$$\frac{\partial^2 \Theta_1}{\partial \tau^2} - a_0^2 \frac{\partial^2}{\partial r^2} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Theta_1}{\partial r} \right) \right]. \tag{13}$$

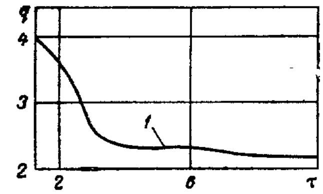

Fig. 3. Time dependence of heat flux: 1) in the critical cross section of the nozzle, q is in kW/cm^2 and τ is in 10^{-1} sec.

Then from (12) it follows directly that

$$\frac{\partial^2\Theta_3}{\partial\tau^2} = a_0 \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left(\frac{\partial\Theta_3}{\partial\tau}\right)\right]. \tag{14}$$

Since six boundary conditions are required to solve Eq. (13) we determined the heat flux in our case by using Eq. (14) which does not require specifying boundary conditions within the interval (R_1, R_2).

Solving Eq. (14) by using the Laplace transform with boundary conditions of the form

$$\Theta_3(r, \tau)|_{\tau=0} = 0 \quad (R_1 \leqslant r \leqslant R_2), \tag{15}$$

$$\left.\frac{\partial\Theta_3(r, \tau)}{\partial\tau}\right|_{\tau=0} = 0 \quad (R_1 \leqslant r \leqslant R_2), \tag{16}$$

$$\Theta_3(r, \tau)|_{r=R_1} = \psi(\tau), \tag{17}$$

$$\Theta_3(r, \tau)|_{r=R_2} = 0, \tag{18}$$

where

$$\psi(\tau) = \frac{1}{\lambda_0}\int_{t_0}^{t}\lambda(t)\,dt + \frac{1}{\rho_0 c_0}\int_{t_0}^{t}\rho_0 c(t)\,dt;$$

and

$$\lambda(t) = \lambda_0 + \lambda_1(t - t_0); \quad c(t) = c_0 + c_1(t - t_0),$$

we obtain

$$\Theta_3(r, \tau) = \left[2(t - t_0) + \frac{1}{2}\left(\frac{\lambda_1}{\lambda_0} + \frac{c_1}{c_0}\right)(t - t_0)^2\right]_{r=R_1}\frac{\ln\frac{R_2}{r}}{\ln\frac{R_2}{R_1}} - \left[2\frac{\partial t}{\partial\tau} + \left(\frac{\lambda_1}{\lambda_0} + \frac{c_1}{c_0}\right)(t - t_0)\frac{\partial t}{\partial\tau}\right]_{r=R_1} \times$$

$$\times \frac{(R_2^2 - r^2)\ln\frac{R_2}{R_1} - (R_2^2 - R_1^2)\ln\frac{R_2}{r} - (r^2 - R_1^2)\ln\frac{R_2}{r}\ln\frac{R_2}{R_1}}{4a_0\ln\frac{2R_2}{R_1}}. \tag{19}$$

In obtaining the solution (19) we limited ourselves to the first derivative of $\psi(\tau)$. We assume that higher-order derivatives are negligibly small.

The heat flux entering the surface is given by

$$q_0 = -\left[\frac{\lambda(t - t_0)\frac{\partial\Theta_3}{\partial r}}{2 + \left(\frac{\lambda_1}{\lambda_0} + \frac{c_1}{c_0}\right)(t - t_0)}\right]_{r=R_1-\Delta}, \tag{20}$$

where $\partial\Theta_3/\partial r$ is found from (19), and $t(r, \tau)_{r=R_1-\Delta}$ from the expression

$$\Theta_3|_{r=R_1-\Delta} = 2(t - t_0)|_{r=R_1-\Delta} + \frac{1}{2}\left(\frac{\lambda_1}{\lambda_0} + \frac{c_1}{c_0}\right)(t - t_0)^2\Big|_{r=R_1-\Delta}. \tag{21}$$

Curve 1 of Fig. 3 shows the time dependence of the heat flux at $r = R_1 - \Delta$. The graph shows that after 0.4 sec a stationary heat flux of the order of 2.4 kW/cm^2 is established.

Thus it follows from the data obtained that using the solution of the nonlinear nonstationary heat-conduction equation to determine heat fluxes gives a unique answer indicating the nature of the heat transfer between the gaseous medium and the surface of the probe.

The use of the linear equation in this case would introduce an error into the estimate of the heat flux, since the thermophysical properties of the probe are temperature dependent.

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